TRINITY COLLEGE

Year 12 Mathematics Specialist Units 3, 4
Test 3 2020

Section 2 Calculator Assumed
Systems of Equations and Vector Calculus

STUDENT'S NAME ____

DATE: Friday 22 May **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Consider the following system of equations. Note: k is a real constant.

$$\begin{aligned} x - 2y + 3z &= 5 \\ x + 2y + z &= 5 \\ 2x - 4y + kz &= 2 \end{aligned}$$

The solutions to the system of equations are:

$x = \dfrac{5k-14}{k-6}$, $y = \dfrac{-4}{k-6}$, $z = \dfrac{A}{k-6}$ where A is a constant.

(a) Explain whether this system of equations has a unique solution for all real values of k. If not, explain the geometric interpretation of this. [3]

(b) Using the third equation, determine the value of A. [3]

2. (10 marks)

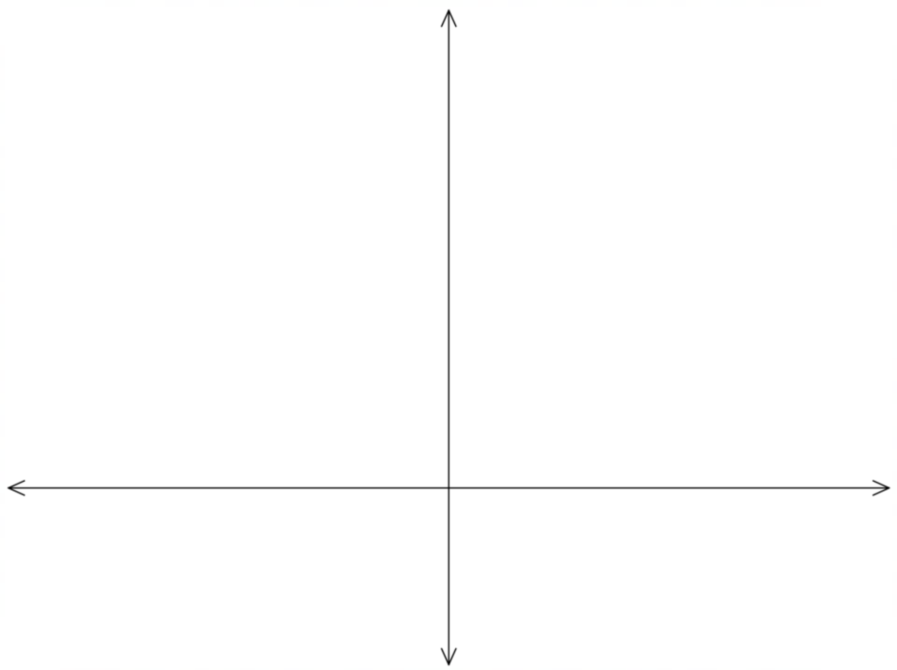
A particle moves such that its position vector is given by $\underset{\sim}{\mathbf{r}}(t) = \begin{pmatrix} t-5 \\ t^2+1 \end{pmatrix}$ for $t \geq 0$.

(a) When does the particle cross the y-axis? [2]

(b) What is the least distance between the particle and the x-axis? [2]

(c) How far is the particle from the origin when $t = 1$? [2]

(d) Sketch the path of the particle showing its direction. [3]



3. (8 marks)

The velocity vector $\underset{\sim}{\mathbf{v}}(t)\ ms^{-1}$ of a particle is given by $\underset{\sim}{\mathbf{v}}(t) = (4\sin 2t)\underset{\sim}{\mathbf{i}} - (3\cos 2t)\underset{\sim}{\mathbf{j}}$.

(a) Determine the displacement vector $\underset{\sim}{\mathbf{r}}(t)$ given that $\underset{\sim}{\mathbf{r}}(0) = -2\underset{\sim}{\mathbf{i}}$. [2]

(b) When will the particle next have a displacement of $-2\mathbf{i}\ m$? [2]

(c) Determine the acceleration vector $\underset{\sim}{\mathbf{a}}(t)$. [2]

(d) Determine the times, in one complete cycle, when the direction of the particle is moving perpendicular to the acceleration vector. [3]

4. (11 marks)

In the Winter Olympics, a figure skater executed a manoeuvre which could best be described as an ellipse. His velocity while completing his part of the routine was given by;

$$\underset{\sim}{\mathbf{v}}(t) = \begin{pmatrix} -6\sin\frac{t}{4} \\ 4\cos\frac{t}{4} \end{pmatrix} ms^{-1}$$

His position on the skating rink at the start of his routine is $\underset{\sim}{\mathbf{r}}(0) = \begin{pmatrix} 24 \\ 0 \end{pmatrix}$.

(a) Determine the position of the skater at time t [2]

(b) If the skater finishes the ellipse when he returns to the starting position, how long has it taken him to complete the manoeuvre? (leave your answer as an exact value) [2]

(c) (i) Prove that the skater's speed can be expressed as

$$Speed = a\sqrt{b - k\cos^2\left(\frac{t}{4}\right)}$$, where a, b and k are positive integer constants.

[4]

(ii) Hence, determine the time(s) at which his speed is the greatest for one complete manoeuvre. State this maximum speed. [3]

5. (15 marks)

The acceleration of a particle projected with speed u at an angle of elevation of α is given by

$$\underset{\sim}{\mathbf{a}}(t)=\begin{pmatrix}0\\-g\end{pmatrix} \text{ with } \underset{\sim}{\mathbf{r}}(0)=\begin{pmatrix}0\\0\end{pmatrix} \text{ and } \underset{\sim}{\mathbf{v}}(0)=\begin{pmatrix}u\cos\alpha\\u\sin\alpha\end{pmatrix}$$

(a) (i) Determine the velocity and displacement vectors $\underset{\sim}{\mathbf{v}}(t)$ and $\underset{\sim}{\mathbf{r}}(t)$. [3]

(ii) Prove that the maximum height attained is $\dfrac{u^2\sin^2\alpha}{2g}$. [3]

(b) A golf ball is struck so that it leaves point A on the ground with speed 49 ms^{-1} at an angle of elevation α. If it lands on the green which is the same level as A, the nearest and furthest points of which are 196 metres and 245 metres respectively from A. (g = 9.8 ms^{-2}).

Given that the horizontal range is given by $\dfrac{u^2 \sin 2\alpha}{g}$

(i) Calculate the set of possible values of α. [3]

(ii) Determine the maximum height the ball can reach and still land on the green. [2]

There is a tree at a horizontal distance 24.5 metres from A and to reach the green the ball must pass over this tree. (The point A, the green and the base of the tree are in the same horizontal plane)

(iii) Determine the maximum height of the tree if this ball can reach any point on the green. [4]